

Estimating Terminal Values with Inflation: The Inputs Matter—It Is Not a Formulaic Exercise

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Estimation of the terminal value of a business enterprise is a critical aspect of any corporate valuation. In a path-breaking article, Bradley and Jarrell showed that traditional constant growth methods for estimating the terminal value typically failed to deal properly with inflation. The premise underlying the Bradley-Jarrell analysis, namely, that inflation applies to a firm's capital stock in the same manner that it applies to other financial metrics, is straightforward. Unfortunately, the Bradley-Jarrell analysis is often misunderstood because of accounting issues that arise in its application and because of its apparent deviation from traditional formulas for plowback and growth. This article addresses those details. In doing so, we demonstrate how to handle the issues that arise when applying the Bradley-Jarrell model to GAAP-based financial statements or forecasts and we show how the Bradley-Jarrell approach can be reconciled with the traditional models.

Introduction

Despite the fact that it typically accounts for the majority of the estimated value of a company, the terminal value in discounted cash flow (DCF) valuations is often treated formulaically without appropriate consideration for the impact of inflation on the inputs. A critical component of terminal valuation estimation is the relationship among growth, plowback, and the return on investment. In an earlier article (Cornell and Gerger 2017) we presented a simple proof of the equation developed by Bradley and Jarrell (2008) that relates the constant growth rate assumed at the terminal horizon to the rate of investment necessary to produce that growth. We also explained why many practitioners have misapplied the “traditional” equation, which states that

$$IR(\text{trad}) = G/\text{RONIC}. \quad (1)$$

In equation 1, $IR(\text{trad})$ is the required reinvestment rate and is stated as a fraction of net operating profit after tax, G is the expected nominal growth rate, and RONIC is the expected nominal return on net new investment. Net operating profit after tax (NOPAT) for purposes of

equation 1 is defined as

$$\text{NOPAT} = \text{IC} \times \text{ROIC}, \quad (2)$$

where, as we discuss further below, IC equals the market value of invested capital and ROIC is the expected nominal return on invested capital. We refer to net operating profit after tax, as defined in equation 2, as $\text{NOPAT}(\text{econ})$. $\text{NOPAT}(\text{econ})$, G , ROIC , and RONIC are economic measures that typically are not derived from a firm's financial statements. When applied to net operating profit after tax derived from GAAP-based financial statements or GAAP-based forecasts, to which we refer as $\text{NOPAT}(\text{acct})$, equation 1 errs by failing to account for the impact of inflation on the existing capital stock. When proper account is taken for inflation, Bradley and Jarrell show that the correct equations to apply to metrics derived from financial statements or forecasts based on GAAP are

$$IR(\text{bj}) = g/\text{ronic}, \quad (3)$$

where g and ronic are stated in real terms (i.e., g is the expected real growth rate and ronic is the expected real return on net new investment) or in nominal terms

$$G = IR(\text{bj}) \times \text{RONIC} + (1 - IR(\text{bj})) \times \pi, \quad (4)$$

where π denotes the steady-state rate of inflation. (Capital letters denote nominal variables, with the exception of inflation.)

One surprising fact is that nearly ten years after its publication, the Bradley-Jarrell equation has yet to be

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adopted universally.¹ This may be due, at least in part, to four accounting complications related to its application. This article addresses all four. They are as follows.

First, Bradley and Jarrell defined the investment rate, $IR(bj)$, net of what they call DET instead of depreciation, which we define as DEP.² DET is defined as the reduction in value of the capital stock over the period of operations. As such, it equals economic depreciation, which also equals the capital expenditures necessary to maintain the firm's capital stock, REP. In conjunction with the distinction between DET and DEP, Bradley and Jarrell introduce a new way to formulate net cash flow, or NCF. In steady state, assuming no real growth, the relation between NOPAT(acct) and NCF is given by

$$NCF = NOPAT(acct) + DEP - REP - \Delta WC(maint), \quad (5)$$

where $\Delta WC(maint)$ denotes the change in working capital necessary to maintain the working capital portion of the firm's capital stock. Equation 5 shows that NCF equals the cash flow available after maintaining the capital stock, but before any capital investment for future growth.³

$IR(bj)$ is then defined as the fraction of NCF that must be plowed back to fund real growth in addition to the maintenance requirements. The 1st issue that arises is that accounting depreciation is rarely equal to REP even if there are no additional working capital requirements. Therefore, REP must be estimated, resulting in the Bradley-Jarrell metrics not lining up with traditional balance sheet and income statement entries. Further, Bradley and Jarrell define NNI as net new investment in excess of REP. This also creates confusion because most practitioners define net new investment as capital expenditures in excess of DEP, not REP.

Second, Bradley and Jarrell define the capital stock, which they denote K , as "the total capital stock at the beginning of the period." As such, it also equals the market value of *all* the firm's capital assets. The firm's

capital stock is frequently denoted as IC, not K (see equation 2). However, IC is equivalent to Bradley and Jarrell's K ; therefore, we will use IC to denote the market value of the firm's total invested capital stock in the discussion below. IC includes the value of internally generated intangibles such as patents, trademarks, customer relationships, etc., that do not appear on a GAAP-based balance sheet but which for a host of firms like Amazon and Google constitute a majority of the market value of the firm's invested capital. By contrast, the capital reported on the balance sheet is based on the historical cost of acquired assets and excludes internally generated intangibles entirely. Although the market value of the firm's invested capital stock does not enter Bradley and Jarrell's final equation for growth, its proper definition is critical for estimating the expected return on investment, ROIC.

Third, Bradley and Jarrell do not provide reconciliations between NCF, NOPAT(acct), and NOPAT(econ), nor do they demonstrate that equations 1 and 4 result in the same growth rate when definitions for the inputs of each equation are carefully applied. These analyses are important because they highlight the differences between NOPAT(econ), NOPAT(acct), and NCF as well as demonstrate that the equation for $IR(trad)$, equation 1, is derived from and should only be applied to NOPAT(econ). In most real-world valuations, NOPAT(econ) does not equal NOPAT(acct) or NCF; therefore, the traditional equation should not be used when the terminal value calculation is based on NOPAT(acct) or NCF.

Fourth, Bradley and Jarrell assume that net new "investment," defined as capital expenditures in excess of REP, is the sole source of future growth. But as Damodaran (2012) emphasizes, many expenses that are included as deductions in forecasts of NOPAT(acct) are the economic equivalent of investments that can finance growth. Two prominent examples are research and development expenses and sales and marketing costs. In addition, the firm may benefit from what we call "efficiencies" that arise during the normal course of business and do not require explicit capital expenditures. These may include learning by doing, taking advantage of exogenously generated technological change, and adopting superior management practices versus competitors.

Taking Account of Accounting Depreciation and the Role of NCF

This section focuses on depreciable assets. We note that for many American corporations, such assets are becoming a smaller fraction of their capital stock that

¹Several recent editions of valuation texts have included specific references to the impact of inflation on constant growth valuation methods: see, for example, Holthausen and Zmijewski (2014) and Brealey, Myers, and Allen (2017); however, many valuation texts fail to consistently differentiate between NOPAT(econ) and NOPAT(acct) when discussing the required plowback investment and constant growth valuation methods.

²In the discussion below, DEP refers to the amount of depreciation reflected on a firm's financial statements, which may differ from the amount of depreciation a firm recognizes for tax purposes.

³We note that equation 5 assumes no real growth because the expenditures for capital and working capital, as well as expenditures on intangibles, are those required to maintain the existing stock of physical capital, working capital, and intangible capital. In this framework, all real growth is funded by plowback, which is deducted from NCF to arrive at FCF.

includes non-depreciable and intangible assets. The impact of that fact is discussed in the next section.

With respect to depreciable assets, the fundamental concept underlying the Bradley-Jarrell analysis is that, in steady state, if the firm maintains its capital stock in real terms, then in nominal terms all the firm's financial metrics, including the capital stock, NOPAT(acct), and free cash flow (FCF), will grow at the rate of inflation. In distinction, real growth requires investment of cash in excess of maintenance requirements. To make that concept clear, Bradley and Jarrell emphasize that their variable DET measures "wear, tear, and obsolescence over the period." This wear, tear, and obsolescence is the economic depreciation of the firm's capital stock.

The accounting issue is that book depreciation, DEP, is typically not equal to DET. DEP only applies to capitalized assets, is based on historical cost, and is based on predetermined depreciation (for capitalized tangible assets) or amortization (for capitalized intangible assets) schedules that likely do not align with the economic productivity of the firm's capital stock. For that reason, NOPAT(acct) typically does not equal cash from operations even if there are no changes in working capital, forcing Bradley and Jarrell to introduce NCF. As shown in equation 5, NCF, in steady state, equals cash from operations after the cost of maintaining the firm's capital stock. FCF is then calculated as NCF minus NNI, as shown in equation 6:

$$FCF = NCF - NNI. \quad (6)$$

With inflation, DEP will tend to be less than DET, because asset prices, like everything else, will rise with inflation. However, the statement that $REP > DEP$ because of inflation assumes that assets are replaced with identical replicas. In fact, this is rarely the case, particularly with regard to technological assets such as computers, for which each successive generation is more productive than the last as a result of innovation. In that case, REP may be less than DEP. Further, if the steady-state rate of innovation equals the rate of inflation, then it is possible that $REP = DEP$ and $NOPAT(acct) = NCF$. Putting those possibilities aside and assuming that assets are replaced by identical replicas, then with inflation it will be the case that $REP > DEP$, so that net capital expenditures (over depreciation) are required to maintain the capital stock.

At 1st blush, the foregoing seems to contradict Bradley and Jarrell. To see why, consider the case in which there is no real growth, only inflation. Equations 3 and 4 both imply that with no real growth $IR = 0$, but if $REP > DEP$, some net capital expenditures are required. The apparent conflict is a result of the failure of book value accounting to reflect the underlying economics.

That is why Bradley and Jarrell had to introduce NCF. When plowback is defined to be investment that is required after provision for maintaining the capital stock, it is clear that no plowback is required if real growth is zero.

It is also worth noting that when $REP > DEP$ as a result of inflation, some reinvestment in excess of depreciation is required to maintain the capital stock, but that in no way rehabilitates equation 1 for most real-world valuations. The issues of how much investment is required to maintain the capital stock and how much is required for real growth are entirely separate, as we clarify further below.

Defining the Capital Stock

Although the capital stock does not directly enter the equations for plowback, its proper definition is still essential. For instance, a starting point for estimating return on investment is the historical relation between NCF and the capital stock. Bradley and Jarrell define K_{t-1} as the total capital stock at the beginning of the period, but they never relate that definition to actual balance sheet measures. Nevertheless, from the context of their analysis it is clear that the capital stock to which they are referring is the market value of *all* the "capital" that goes into the production of NCF (i.e., the capital stock is the market value of the firm's total invested capital, including debt, equity, and intangibles). This is quite different than the capital stock that appears on GAAP-based balance sheets. Balance sheet assets under GAAP consist primarily of the depreciated value of the historical cost of depreciable and non-depreciable tangible assets and the cost of acquired or capitalized intangible assets net of any amortization. Internally generated intangible capital (e.g., the value of internally generated customer relationships, research and development, patents, etc.) is often excluded. There is thus a potentially huge gap between what Bradley and Jarrell, and most of the valuation literature, call capital and what can be read off the balance sheet. Inflation tends to increase the gap by driving a wedge between historical cost and the current value of capitalized assets.

The gap between book capital and total capital can have a dramatic impact on estimated ROIC from GAAP-based accounting statements. This is why an estimate of the market value of the total capital stock is required to implement equation 3 or 4.

Capital Stock, Inflation, and NOPAT(econ)

The market value of the firm's total invested capital is also necessary to determine NOPAT(econ). As

shown in equation 2, NOPAT(econ) is typically defined as $IC \times ROIC$.⁴ Further, ROIC in equation 2 is equal to the expected nominal return on invested capital that can be redefined based on the expected real return on invested capital (roic) and inflation. The relation is given by

$$ROIC = (1 + roic) \times (1 + \pi) - 1, \quad (7)$$

or

$$ROIC = roic \times (1 + \pi) + \pi. \quad (8)$$

Substituting equation 8 for ROIC in equation 2 produces

$$NOPAT(econ) = IC \times (roic \times (1 + \pi) + \pi), \quad (9)$$

or

$$NOPAT(econ) = IC \times roic \times (1 + \pi) + IC \times \pi. \quad (10)$$

The final term in equation 10 shows that NOPAT(econ) includes the impact of inflation on the firm's existing capital stock. The inclusion of appreciation on a firm's capital stock due to inflation typically results in a difference between NOPAT(econ) and NOPAT(acct). Further, because equation 1 is based on NOPAT(econ), it should not be applied to NCF or NOPAT(acct).

As shown in equation 5, in steady state, NCF can be defined in terms of NOPAT(acct), DEP, REP, and $\Delta WC(maint)$. Bradley and Jarrell show that, in steady state, NCF can also be defined in terms of IC and roic, such that

$$NCF = IC \times roic \times (1 + \pi). \quad (11)$$

Substitution of NCF from equation 11 into equation 10 allows NOPAT(econ) to be expressed in terms of NCF, IC, and inflation. The relation is given by

$$NOPAT(econ) = NCF + IC \times \pi. \quad (12)$$

Again, the final term in equation 12 represents the impact of inflation on the firm's existing capital stock.

Substitution of equation 5 for NCF in equation 12 allows NOPAT(econ) to be expressed in terms of NOPAT(acct). The relation is given by

$$NOPAT(econ) = NOPAT(acct) + DEP - REP - \Delta WC(maint) + IC \times \pi. \quad (13)$$

⁴Notice that as it is defined, NOPAT(econ) is not of much practical use in valuation. In order to calculate NOPAT(econ), it is necessary to know the market value of all IC, including intangibles. But if the market value of IC is known, so is the enterprise value of the firm, which makes information regarding NOPAT(econ) redundant.

From equation 13, it is evident that NOPAT(econ) equals NOPAT(acct) when $REP - DEP - \Delta WC(maint) = IC \times \pi$. Assuming $\Delta WC(maint)$ is zero, it can be demonstrated that $REP - DEP = IC \times \pi$ when a firm's invested capital consists of only depreciable assets and clean-surplus accounting applies and when assets are replaced with identical replacements. If *any* of these conditions fail (i.e., the firm's invested capital includes intangible assets or non-depreciable assets), then $REP - DEP \neq IC \times \pi$ and $NOPAT(econ) \neq NOPAT(acct)$, even if there is no change in working capital necessary to maintain the firm's capital stock. Furthermore, equation 13 makes it clear that NOPAT(econ) is an inclusive concept. As a true economic measure, it incorporates both the impact of inflation on the capital stock and possible differences between depreciation and replacement cost. The problem with NOPAT(econ) is that it cannot be derived directly from a firm's financial statements. Where valuation analysts go astray is in substituting NOPAT(acct) for NOPAT(econ) without taking proper account of inflation, as Bradley and Jarrell emphasize.

Equation 13 also reveals that the inflationary increase in the firm's capital stock must be deducted from NOPAT(econ) to arrive at FCF. This relation can be clearly established by substituting equation 5 for NCF in equation 6:

$$FCF = NOPAT(acct) + DEP - REP - \Delta WC(maint) - NNI, \quad (14)$$

or

$$FCF + NNI = NOPAT(acct) + DEP - REP - \Delta WC(maint). \quad (15)$$

Substituting equation 14 into equation 13 and rearranging terms results in

$$NOPAT(econ) = FCF + NNI + IC \times \pi, \quad (16)$$

or

$$FCF = NOPAT(econ) - NNI - IC \times \pi. \quad (17)$$

It is generally accepted that in steady state, the terminal value can be calculated using FCF, thus:

$$\text{Terminal Value} = \frac{FCF_{(t+1)}}{W - G}, \quad (18)$$

where $FCF_{(t+1)}$ equals the normalized level of FCF in the 1st year of the terminal period, W equals the weighted average cost of capital, and G equals the nominal rate of growth in perpetuity.

It is also generally accepted that in steady state, the terminal value can be calculated using the value driver

formula based on NOPAT(econ) and the traditional reinvestment rate from equation 1:

$$\text{Terminal Value} = \frac{\text{NOPAT}(\text{econ})_{(t+1)} \times (1 - G/\text{RONIC})}{W - G}, \quad (19)$$

where $\text{NOPAT}(\text{econ})_{(t+1)}$ equals the normalized level of NOPAT(econ) in the 1st year of the terminal period, W equals the weighted average cost of capital, G equals the nominal rate of growth in perpetuity, and RONIC equals the nominal return on net new invested capital.

Based on equation 1, equation 19 can be restated in terms of IR(trad), thus:

$$\text{Terminal Value} = \frac{\text{NOPAT}(\text{econ})_{(t+1)} - \text{NOPAT}(\text{econ})_{(t+1)} \times \text{IR}(\text{trad})}{W - G}. \quad (20)$$

A comparison of the numerators in equations 18 and 20 shows that in steady state the relation between FCF and NOPAT(econ) can also be expressed as

$$\text{FCF} = \text{NOPAT}(\text{econ}) - \text{NOPAT}(\text{econ}) \times \text{IR}(\text{trad}). \quad (21)$$

From equations 17 and 21 it can be seen that

$$\text{NOPAT}(\text{econ}) \times \text{IR}(\text{trad}) = \text{NNI} + \text{IC} \times \pi. \quad (22)$$

Equation 22 shows that the traditional investment rate formula results in the deduction of an amount equal to the impact of inflation on the firm's capital stock in addition to NNI. The traditional investment rate formula, equation 1, is appropriate, but *only* when applied to NOPAT(econ), because as shown in equation 10, NOPAT(econ) incorporates the impact of inflation on the firm's capital stock. However, the traditional investment rate formula cannot be applied to NOPAT(acct) or other GAAP-based metrics, which typically do not include the impact of inflation on the firm's capital stock.

It is also important to note that when both approaches, that using NOPAT(acct) and that using NOPAT(econ), are applied consistently, they lead to the same measure of FCF in the terminal year. This means they result in the same estimate of the terminal value.

The critical distinction between NOPAT(econ) and NOPAT(acct) is often ignored in valuation texts. For example, in a widely cited book, Koller, Goedhart, and Wessels (2016) derive several equations, including equation 1, using NOPAT(econ), but then in their valuation examples they apply the results using NOPAT(acct). As Bradley and Jarrell emphasize, this leads to incorrect results.

Several articles have claimed to show that Bradley and Jarrell's formula for growth is equivalent to the traditional formula for growth.⁵ As demonstrated in Appendix A, this observation is correct as long as the traditional investment rate is applied to NOPAT(econ). If the valuation analysis is based on accounting-based financial statements, then Bradley and Jarrell's investment rate must be used and applied to NCF. To conflate the two is to make the same error as Koller, Goedhart, and Wessels. As a practical matter, because NOPAT(econ) is not observable, appraisers are typically forced to rely on accounting measures. In such circumstances, the correct approach is to use the Bradley-Jarrell plowback equation in conjunction with NCF.

Sources of Growth Other Than Capital Expenditure

The Bradley-Jarrell equations are derived under the assumption that capital expenditures are the only source of real growth. If standard accounting conventions are used to define capital expenditures, this is counterfactual. As Damodaran (2010, 2012) observes, many items that are expensed on the income statement are capital expenditures in disguise. Research and development and advertising are two examples. This complicates the calculation of IR. One potential solution to the problem is to continue to use the Bradley-Jarrell equation, but to capitalize expenditures that are effectively investments that serve to produce future growth. Of course, that requires a financial forecast sufficiently granular to identify such expenditures.

In addition, as noted earlier, growth can arise from efficiencies that arise during the normal course of business. Any costs associated with these efficiencies would be embedded in the financial projections and would not require added capital expenditure.

There is a related issue with respect to the maintenance of intangible capital. With regard to depreciable capital, Bradley and Jarrell make explicit allowance for the costs necessary to maintain the capital stock. But what about the costs of maintaining intangible capital, such as Amazon's goodwill with its customers? Unlike the replacement of depreciable assets, the expenditures necessary to maintain intangibles are generally buried in the projection of the income statement. This makes it particularly important for a valuation analyst to understand what goes into the financial projections in the years prior to the terminal value. The key question is "To what extent do those projections include expenditures for

⁵See Friedl and Schwetzler (2008), Jennergren (2011), and Kiechle and Lampenius (2012a, 2012b).

maintaining, or even enhancing, the stock of intangible capital?”

Finally, we note that despite the attention paid to plowback in the academic literature, the issue is often ignored in practitioner valuation. For instance, when the projected real growth rates are low, we have found that valuations prepared by both investment banks and companies involved in transactions often provide for minimal capital expenditure at the terminal horizon and rarely apply either the traditional model or the Bradley-Jarrell model explicitly. While one hypothesis is that this is simply an oversight on the part of practitioners, their behavior may in fact be rational. When real growth rates are low, it is quite possible that they can be funded entirely by items that are run through the income statement and by taking advantage of efficiencies.

Conclusions and Implications for Valuation Analysis

As a practical matter, valuations are almost invariably performed using both historical accounting data and projections of future accounting statements. Therefore, concepts like NOPAT(econ), though useful in theory, are rarely employed in practice. In light of that, the results developed here have several important implications for estimation of the terminal value.

First, to calculate the plowback required for growth, the appropriate procedure is to use the Bradley-Jarrell equation applied to NCF. Second, computation of NCF requires a careful analysis of the capital expenditure, working capital expenditure, and other expenditures required to maintain the productive capacity of the capital stock, including intangibles. These calculations can be complicated because they are affected by inflation and technical change, among other things. If the simplifying assumptions are made that maintenance capital expenditure equals depreciation, there is no change in the firm's working capital requirement, and other projected expenditures are sufficient to maintain the firm's intangible capital stock, then $NCF = NOPAT(acct)$, and the Bradley-Jarrell equation can be applied directly to $NOPAT(acct)$. Third, the expected return on net new invested capital, RONIC, used in the Bradley-Jarrell equation, must be an economic return, not an accounting return, as measured by $NOPAT(acct)$ divided by accounting assets. This can be difficult to estimate because it depends on returns generated on the new capital stock, including the ability to continue to internally generate intangibles that do not appear on the balance sheet. Finally, the possibility that growth is due to causes other than capital expenditures must be addressed. One alternative source of growth is

expenditures run through the income statement, such as advertising and research and development. In addition, growth can also be due to the efficiencies that arise in the normal course of business. If these sources of growth are important and if they are included in the calculation of $NOPAT(acct)$, the required investment rate to achieve a given level of growth may be less than that given by the Bradley-Jarrell equation.

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Appendix A

The traditional equation for growth is expressed as

$$G = \text{IR}(\text{trad}) \times \text{RONIC}, \quad (1)$$

where IR(trad) can be expressed as

$$\text{IR}(\text{trad}) = \frac{\text{NOPAT}(\text{econ}) - \text{FCF}}{\text{NOPAT}(\text{econ})}. \quad (2)$$

ROIC, the nominal return on invested capital, can be expressed as

$$\text{ROIC} = (1 + \text{roic})(1 + \pi) - 1, \quad (3)$$

or

$$\text{ROIC} = \text{roic} + \text{roic} \times \pi + \pi, \quad (4)$$

where roic is the real return on invested capital and π is the rate of inflation.

Further, NOPAT(econ) is equal to

$$\text{NOPAT}(\text{econ}) = \text{IC} \times \text{ROIC}, \quad (5)$$

where IC is equal to the firm's capital stock.

Combining equations 1 and 2 results in

$$G = \frac{\text{NOPAT}(\text{econ}) - \text{FCF}}{\text{NOPAT}(\text{econ})} \times \text{RONIC}. \quad (6)$$

Substituting equation 5 for NOPAT(econ) yields

$$G = \frac{\text{IC} \times \text{ROIC} - \text{FCF}}{\text{IC} \times \text{ROIC}} \times \text{RONIC}. \quad (7)$$

For simplicity, we assume ROIC equals RONIC in steady state, although we note this may not be appropriate in real-world appraisals.⁶ With this assumption, equation 7 can be simplified to

$$G = \frac{\text{IC} \times \text{ROIC} - \text{FCF}}{\text{IC}}. \quad (8)$$

Substituting equation 4 for ROIC in equation 8 results in

$$G = \frac{\text{IC} \times (\text{roic} + \text{roic} \times \pi + \pi) - \text{FCF}}{\text{IC}}. \quad (9)$$

Rearranging terms results in

$$G = \frac{\text{IC} \times (\text{roic} + \text{roic} \times \pi) - \text{FCF}}{\text{IC}} + \frac{\text{IC} \times \pi}{\text{IC}}, \quad (10)$$

which can be simplified to

$$G = \frac{\text{IC} \times (\text{roic} + \text{roic} \times \pi) - \text{FCF}}{\text{IC}} + \pi. \quad (11)$$

Bradley and Jarrell's equation for growth is

$$G = \text{IR}(\text{bj}) \times \text{RONIC} + (1 - \text{IR}(\text{bj})) \times \pi, \quad (12)$$

where IR(bj) can be expressed as

$$\text{IR}(\text{bj}) = \frac{\text{NCF} - \text{FCF}}{\text{NCF}} \quad (13)$$

and NCF can be expressed as

$$\text{NCF} = \text{IC} \times (\text{roic} + \text{roic} \times \pi). \quad (14)$$

Combining equations 12 and 13 results in

$$G = \frac{\text{NCF} - \text{FCF}}{\text{NCF}} \times \text{RONIC} + \pi - \frac{\text{NCF} - \text{FCF}}{\text{NCF}} \times \pi. \quad (15)$$

Again, for simplicity we assume ROIC equals RONIC in steady state. With this assumption, substituting equation 4 for RONIC in equation 15 results in

$$G = \frac{\text{NCF} - \text{FCF}}{\text{NCF}} \times (\text{roic} + \text{roic} \times \pi + \pi) + \pi - \frac{\text{NCF} - \text{FCF}}{\text{NCF}} \times \pi. \quad (16)$$

Rearranging terms results in

$$G = \frac{\text{NCF} - \text{FCF}}{\text{NCF}} \times (\text{roic} + \text{roic} \times \pi) + \frac{\text{NCF} - \text{FCF}}{\text{NCF}} \times \pi + \pi - \frac{\text{NCF} - \text{FCF}}{\text{NCF}} \times \pi. \quad (17)$$

Equation 17 can be simplified to

$$G = \frac{\text{NCF} - \text{FCF}}{\text{NCF}} \times (\text{roic} + \text{roic} \times \pi) + \pi. \quad (18)$$

Substituting equation 14 for NCF in equation 18 yields

$$G = \frac{\text{IC}(\text{roic} + \text{roic} \times \pi) - \text{FCF}}{\text{IC}(\text{roic} + \text{roic} \times \pi)} \times (\text{roic} + \text{roic} \times \pi) + \pi, \quad (19)$$

which can be simplified to

$$G = \frac{\text{IC}(\text{roic} + \text{roic} \times \pi) - \text{FCF}}{\text{IC}} + \pi. \quad (20)$$

Equation 20 equals equation 11, derived using the traditional investment rate formula and NOPAT(econ).

⁶This assumption does not alter the results of this appendix.